

A cat jumps from a window of a building, at an angle of 53.13° above the horizontal, with an initial speed of 7.5 feet per second, and lands safely on the ground 9 feet away from the base of the building. Let h be the height of the window above ground level.

SCORE: ____ / 25 PTS

NOTE: $\sin 53.13^\circ = \frac{4}{5}$ and $\cos 53.13^\circ = \frac{3}{5}$

[a] Write parametric equations for the cat's position.

$$x = (7.5 \cos 53.13^\circ)t = 4.5t \quad (4)$$

$$y = h + (7.5 \sin 53.13^\circ)t - 16t^2 = h + 6t - 16t^2 \quad (4)$$

[b] How high off the ground is the window?

$$x = 4.5t = 9 \quad (3)$$

$$t = 2 \quad (2)$$

$$y = h + 6(2) - 16(2)^2 = 0 \quad (3) \quad \text{GROUND LEVEL}$$

$$h = 52 \text{ FEET} \quad (2)$$

Using mathematical induction, prove that $2^3 + 4^3 + \dots + (2n)^3 = 2n^2(n+1)^2$ for all positive integers n . SCORE: ____ / 25 PTS

BASIS STEP: $2^3 = 8$ (2) $2(1)^2(1+1)^2 = 2(1)(4) = 8$ (2)

INDUCTIVE STEP: ASSUME $2^3 + 4^3 + \dots + (2k)^3 = 2k^2(k+1)^2$ FOR SOME (2)
PARTICULAR BUT ARBITRARY INTEGER $k \geq 1$

(3) $2^3 + 4^3 + \dots + (2k)^3 + (2(k+1))^3$

(3) $= 2k^2(k+1)^2 + (2(k+1))^3$

$= 2(k+1)^2 [k^2 + 4(k+1)]$ (5)

$= 2(k+1)^2 (k^2 + 4k + 4)$

(3) $= 2(k+1)^2 (k+2)^2$

BY MI, $2^3 + 4^3 + \dots + (2n)^3 = 2n^2(n+1)^2$ FOR ALL INTEGERS $n \geq 1$ (2)

Consider the expression $(9x^8 - 7x^5)^{27}$.

SCORE: ____ / 25 PTS

- [a] Write the first 3 terms of the expansion of the expression. Simplify all exponents.

Your answer may use multiplication and exponents, but NOT division, ! nor ${}_nC_r$ (or equivalent) notation.

$$\begin{aligned}
 & \binom{27}{0}(9x^8)^{27} + \binom{27}{1}(9x^8)^{26}(-7x^5) + \binom{27}{2}(9x^8)^{25}(-7x^5)^2 \\
 &= \frac{27!}{0!27!} 9^{27} x^{216} + \frac{27!}{1!26!} 9^{26} (-7) x^{213} + \frac{27!}{2!25!} 9^{25} (-7)^2 x^{210} \\
 &= 9^{27} x^{216} - 27 \cdot 9^{26} \cdot 7 x^{213} + \frac{27 \cdot 26}{2} 9^{25} 7^2 x^{210} \\
 &= \underbrace{9^{27}} \underbrace{x^{216}} - \underbrace{27 \cdot 9^{26} \cdot 7}_{(2)} \underbrace{x^{213}} + \underbrace{27 \cdot 13}_{(2)} \underbrace{9^{25}} \underbrace{7^2}_{(2)} \underbrace{x^{210}}
 \end{aligned}$$

① POINT
EXCEPT AS NOTED

- [b] Find the coefficient of x^{93} in the expansion.

Your answer may use multiplication, division, exponents and !, but NOT ${}_nC_r$ (or equivalent) notation.

$$\begin{aligned}
 & \binom{27}{r}(9x^8)^{27-r}(-7x^5)^r \\
 &= \frac{27!}{r!(27-r)!} 9^{27-r} (-7)^r x^{216-3r}
 \end{aligned}$$

⑥ $216 - 3r = 93$

$-3r = -123$

② $r = 41 > 27$

② NO SUCH TERM (x^{93})

BONUS ⑤ COEFFICIENT = 0

OR

⑩ LOWEST POWER
OF x IS $(x^5)^{27}$
 $= x^{135}$

Simplify $\frac{(3n-2)!}{(3n+1)!}$.

SCORE: ____ / 10 PTS

$$\frac{\cancel{(3n-2)!}}{\underbrace{(3n+1)(3n)(3n-1)\cancel{(3n-2)!}}_{\textcircled{6}}} = \frac{1}{\underbrace{3n(3n+1)(3n-1)}_{\textcircled{4}}}$$

Find the value of $\sum_{n=2}^{\infty} 500(0.8)^{2n-1}$. HINT: Write out the first few terms first.

SCORE: ____ / 15 PTS

$$500(0.8)^3 + 500(0.8)^5 + 500(0.8)^7 + \dots$$

$$S = \frac{\overbrace{4}^{(4)} * (0.8)^2 \overbrace{500(0.8)^3}^{(4)}}{1 - \underbrace{(0.8)^2}_{(3)}} = \underbrace{711.\bar{1}}_{(4)}$$

Use sigma notation to write the series

$$-\frac{231}{37} + \frac{217}{54} - \frac{203}{71} + \frac{189}{88} - \dots + \frac{21}{292}$$

$\begin{array}{ccccccc} & -14 & -14 & -14 & & & \\ & \curvearrowright & \curvearrowright & \curvearrowright & & & \\ & +17 & +17 & +17 & & & \end{array}$

SCORE: ____ / 15 PTS

$$\sum_{n=1}^{16} (-1)^n \frac{231 - 14(n-1)}{37 + 17(n-1)}$$

(2) above 16
 (2) below n=1
 (2) below (-1)ⁿ
 (3 1/2) next to 231 - 14(n-1)
 (3 1/2) next to 37 + 17(n-1)

$$37 + 17(n-1) = 292$$

$$17(n-1) = 255$$

$$n-1 = 15$$

$$n = 16$$

(2) next to the first equation

(-2) IF INDEX DOESN'T MATCH
INSIDE OF SUMMATION

Eliminate the parameter to find a rectangular equation corresponding to the parametric equations

$$x = 4 + 5 \sec t$$

$$y = 3 + 7 \tan t$$

SCORE: ____ / 15 PTS

For your final answer, write y as a **simplified** function of x .

$$\sec t = \frac{x-4}{5} \quad (4)$$

$$\tan t = \frac{y-3}{7} \quad (4)$$

$$\sec^2 t - \tan^2 t = 1$$

$$\left(\frac{x-4}{5}\right)^2 - \left(\frac{y-3}{7}\right)^2 = 1 \quad (7)$$

For the month of October 2013, MJ's and NJ's water bills were each \$23. Over the next two years, MJ's monthly bill has increased 25 cents each month, while NJ's monthly bill has increased 1% each month. Whose total water bills over the last two years has been higher, and by how much? SCORE: ____ / 20 PTS

$$\text{MJ: } S_{24} = \frac{24}{2} (2(23) + (0.25)(24-1)) = 621$$

$$\text{NJ: } S_{24} = \frac{23(1.01^{24}-1)}{1.01-1} = 620.39$$

MJ's BILLS TOTALLED 61 CENTS MORE THAN NJ's

(2) POINTS EXCEPT AS NOTED